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Enhanced excitation of giant pairing vibrations in heavy-ion reactions induced by weakly bound projectiles

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Abstract. The use of radioactive ion beams is shown to offer the possibility to study collective pairing states at high excitation energy, which are not usually accessible with stable projectiles because of large energy mismatch. In the case of two-neutron stripping reactions induced by ^{6}He, we predict a population of the giant pairing vibration in ^{208}Pb or ^{116}Sn with cross-sections of the order of a millibarn, dominating over the mismatched transition to the ground state.

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1 Introduction

Large efforts have been recently dedicated to the study of different aspects of reaction mechanism in collisions induced by weakly bound radioactive beams. The long tails of the one-particle transfer form factors due to the weak binding, associated with the possibility of unusual behaviour of pairing interaction in diluted systems, has raised novel interest in the possibility of studying the pair field via two-particle transfer processes with unstable beams [1]. On the other hand, in transfer reactions induced by weakly bound projectiles on stable targets, the Q-values for the low-lying states will be very large (typically of the order of 10–15 MeV for the (^{6}He,^{4}He) stripping reaction). This will strongly hinder these processes for reactions where the semi-classical optimum matching conditions apply, as it is the case of bombarding energies around the Coulomb barrier on heavy target nuclei. Higher bombarding energies, where the matching conditions are less stringent, may on the other hand not be suitable because of large break-up cross-sections. The same matching conditions will favour instead the population of highly excited states, as the giant pairing vibrations (GPV), and the use of radioactive ion beams (RIB) may therefore become instrumental in offering the opportunity of studying nuclear-structure aspects that are not usually accessible with stable projectiles. These giant pairing vibrations are in fact predicted [2] to have strong collective features, but their observation may have so far failed [3] because of large mismatch in reactions induced by protons or tritons, at variance to the case of the low-lying pairing vibrations, which have been intensively and successfully studied around closed-shell nuclei in two-particle transfer reactions [4]. All these 0^+ states are associated with vibrations of the Fermi surface and are described in a microscopic basis of the shell model as correlated two-particle–two-hole states. In the case of the giant pairing vibrations the excitation involves the promotion of a pair of particles (or holes) in the next major shell (hence an excitation energy around $2\hbar\omega$) and is expected to display a collective pairing strength comparable with the low-lying vibrations. Also in the case of superfluid systems in an open shell the system is expected to display a collective high-lying state, that in this case collects its strength from the unperturbed two-quasiparticle 0^+ states with energy $2\hbar\omega$. To investigate this possibility we made estimates of cross-sections to the giant pairing vibrations in two-particle transfer reactions, comparing the cases of bound or weakly bound projectiles. As examples we have considered the case of (^{14}C,^{12}C), from one side, and the case of (^{6}He,^{4}He) as representative of a reaction induced by a weakly bound ion. As targets, we have chosen the popular cases of the lead and tin regions (so considering both "normal" and "superfluid" nuclei). To perform the calculation, we will first evaluate the response to the pairing operator in the RPA, including both the low-lying and high-lying pairing vibrations. As a following step we will then construct two-neutron transfer form factors, using the "macroscopic" model for pair transfer processes. Finally, estimates of cross-sections will be given using standard DWBA techniques. As we will see, in the case of the stripping reaction induced by ^{6}He,

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the population of the GPV is expected to display cross-sections of the order of a millibarn, dominating over the mismatched transition to the ground state.

The paper is organized as follows. In the next section we discuss the theoretical formalism used for normal and for superfluid nuclei. In sect. 3 we recall the basics aspects of the macroscopic form factors for two-particle transfer reactions and in sect. 4 we display the results of calculations for the paradigmatic examples of ^{208}Pb and ^{116}Sn.

2 The pairing response and the giant pairing vibration

A simple way of displaying the amount of pairing correlations is in terms of the pair transfer transition densities [5]. These are defined as the matrix element of the pair density operator connecting the ground state in nucleus A with the generic 0^+ state $|n\rangle$ in nucleus $A \pm 2$, namely

$$\delta\rho_{\mathrm{P}}(r) = \langle n|\hat{\rho}_{\mathrm{P}}|0\rangle, \tag{1}$$

where the generalized density operator is given by

$$\hat{\rho}_{\mathrm{P}}(r) = \sum_{\alpha} \frac{\sqrt{2j+1}}{4\pi} R_{\alpha}(r) R_{\alpha}(r) ([a^{\dagger}_{\alpha} a^{\dagger}_{\alpha}]_{00} + [a_{\alpha} a_{\alpha}]_{00}). \tag{2}$$

Here $R_{\alpha}(r)$ are the radial wave functions of the $\alpha = \{nlj\}$ level and the sum runs over both particle and hole levels. The pair transfer strength to each final state can be obtained from the corresponding pair transfer transition density by simple quadrature, namely

$$\beta_{\mathrm{P}} = \int 4\pi r^2 \delta\rho_{\mathrm{P}} \mathrm{d}r. \tag{3}$$

For normal systems around closed shell the strong $L = 0$ transition follows a vibrational scheme, where the correlated pair of fermions (pairing phonon) change by one [6]. In this case, there are two types of phonons associated with the stripping and pick-up reactions. The two-particle collective state is called “addition” pairing phonon while the two-holes correlated state is known as “removal” pairing phonon. From a microscopic point of view the two kind of phonons, corresponding to the $(A \pm 2)$ nuclei can be described in terms of the two-particle (two-hole) states of the Tamm-Dancoff approximation (TDA) or in a better way by a random phase approximation (RPA). We start from an Hamiltonian with a monopole pairing interaction

$$H = \sum_{j} \epsilon_j a^{\dagger}_j a_j - G 4\pi P^{\dagger} P, \tag{4}$$

where

$$P^{\dagger} = \sum_{j_1 \le j_2} \frac{M(j_1, j_2)}{\sqrt{1+\delta_{j_1 j_2}}} \left[a^{\dagger}_{j_1} a^{\dagger}_{j_2} \right]_{00}. \tag{5}$$

Here the $a^{\dagger}_j$ creates a particle in an orbital j, where j stands for all the needed quantum numbers of the level. The constant G is the strength of the pairing interaction and the coefficients $M(j_1, j_2)$ are:

$$M(j_1, j_2) = \frac{\langle j_1 || f(r) Y_{00}(\theta, \phi) || j_2 \rangle}{\sqrt{1+\delta_{j_1 j_2}}}, \tag{6}$$

where the detailed radial dependence of $f(r)$ is taken to be of the form r^L and in our case is a constant since we are dealing only with $L = 0$ states. The pairing phonons are defined for closed-shell nuclei as

$$\begin{aligned} |n, 2p\rangle &= \Gamma^{\dagger}_{n,2p}|0\rangle_{\mathrm{RPA}} = \\ &\left(\sum_k X_n(k) [a^{\dagger}_k a^{\dagger}_k]_{00} + \sum_i Y_n(i) [a^{\dagger}_i a^{\dagger}_i]_{00} \right) |0\rangle_{\mathrm{RPA}} \\ |n, 2h\rangle &= \Gamma^{\dagger}_{n,2h}|0\rangle_{\mathrm{RPA}} = \Gamma_{n,2p}|0\rangle_{\mathrm{RPA}} = \\ &\left(\sum_i X_n(i) [a_i a_i]_{00} + \sum_k Y_n(k) [a_k a_k]_{00} \right) |0\rangle_{\mathrm{RPA}}, \end{aligned} \tag{7}$$

where $k(i)$ stands for levels above (below) the Fermi level. The index j runs over both particle and hole levels. We have indicated with $|0\rangle_{\mathrm{RPA}}$ the correlated RPA vacuum. It represents the ground state with respect to the boson annihilation operator $\Gamma^{\dagger}_{n,2h}|0\rangle_{\mathrm{RPA}} = 0$. The definitions of X_n and Y_n (called forward and backward amplitudes) are the standard ones and come from the solution of the RPA equation. They may be found in [6]. Within this model the pair transfer strength associated with each RPA state is microscopically given by

$$\beta_{\mathrm{P}n} = \sum_j \sqrt{2j+1} [X_n(j) + Y_n(j)]. \tag{8}$$

In fig. 1a we display the predicted pairing response in the case of ^{206}Pb, namely two-neutron holes with respect to the double magic ^{208}Pb. The set of single-particle levels that has been used in the RPA calculation, was obtained using the spherical harmonic-oscillator levels with corrections due to the centrifugal and spin-orbit interactions [7]

$$\frac{E}{\hbar\omega} = N + \frac{3}{2} - \mu \left(l(l+1) - \frac{N(N+3)}{2} \right) + K\,, \tag{9}$$

$$K = \begin{cases} -\kappa l\,, & \text{for } j = l + 1/2\,, \\ -\kappa(l+1)\,, & \text{for } j = l - 1/2\,, \end{cases}$$

where $\hbar\omega = 41 A^{-\frac{1}{3}}$, A is the mass number of the nucleus, N is the principal quantum number and j, l are the total and orbital angular momentum quantum numbers, respectively. The quantities κ and μ are parameters chosen to obtain the best fit for each nucleus [8]. We have included in the calculation all the single-particle levels starting from $N = 0$ up to 10. This set is expected to be good enough for our calculation of the giant pairing resonance, except for the levels around the Fermi surface. In the lead region we prefer to use experimental values for the shells just above and below the Fermi surface [9,10].

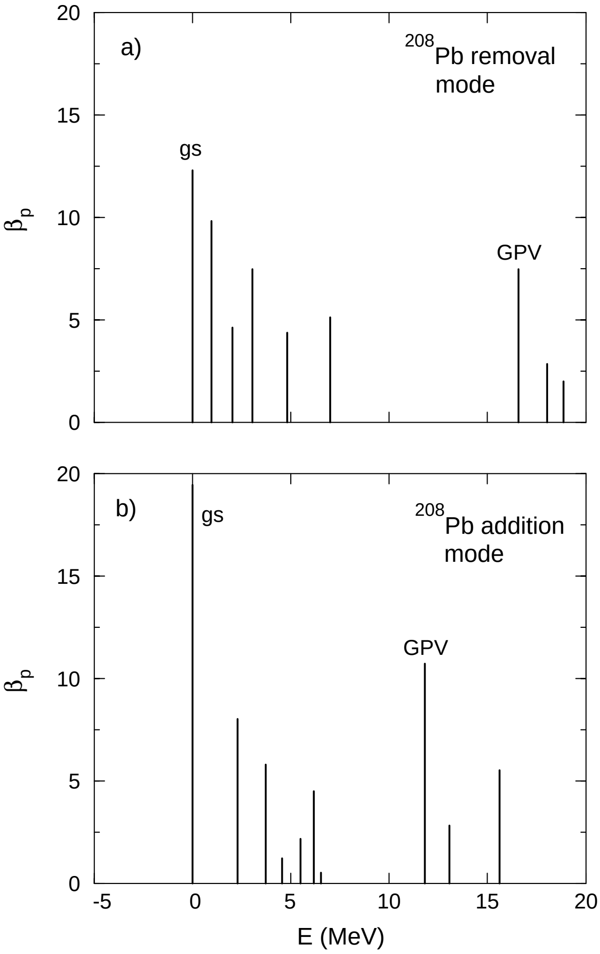

Fig. 1. Pairing response for the removal (a) and addition (b) mode in ^{208}Pb. The ground-state transition and the candidate for GPV are evidenced.

Figure 1a shows, in addition to the strong collectivity associated with the ground-state transition, a strong collective state with about half of the g.s. strength at high excitation energy, around 16 MeV, which can be interpreted as the giant pairing vibration. Similar situation is shown in fig. 1b for the corresponding two-neutron addition states in the ^{210}Pb. Again one may interpret the strength at about 12 MeV as associated with the giant mode. Note that in both addition and removal cases, the contribution of the backward amplitudes to the wave function is found to be roughly equivalent to 5–10% in the ground state, while in the GPV this contribution reduces to less than 1%.

We consider now the case of superfluid spherical nuclei. In this case we make a BCS transformation of the Hamiltonian defined in eq. (4) changing from particle to quasiparticle operators, introducing the usual occupation parameters. We start from a single-quasiparticle Hamiltonian plus a two-quasiparticle interaction corresponding to the residual $H_{22}+H_{40}$ of the pairing force

$$H = \sum_j E_j \alpha_j^\dagger \alpha_j + 2\pi G \sum_{j_1 j_2} M(j_1,j_1) M(j_2,j_2)$$

$$\cdot \Big\{ (U_{j_1}^2 U_{j_2}^2 + V_{j_1}^2 V_{j_2}^2) [\alpha_{j_1}^\dagger \alpha_{j_1}^\dagger]_{00} [\alpha_{j_2}\alpha_{j_2}]_{00} - U_{j_1}^2 V_{j_2}^2 [\alpha_{j_1}^\dagger \alpha_{j_1}^\dagger]_{00} [\alpha_{j_2}^\dagger \alpha_{j_2}^\dagger]_{00} - V_{j_1}^2 U_{j_2}^2 [\alpha_{j_1}\alpha_{j_1}]_{00} [\alpha_{j_2}\alpha_{j_2}]_{00} \Big\}, \quad (10)$$

where

$$\alpha_j^\dagger = U_j a_j^\dagger - V_j a_{\bar{j}}\,, \quad (11)$$

$$U_j^2 = \frac{1}{2}\left(1 + \frac{\tilde{\epsilon}_j}{E_j}\right), \quad (12)$$

$$V_j^2 = \frac{1}{2}\left(1 - \frac{\tilde{\epsilon}_j}{E_j}\right). \quad (13)$$

The energies $E_j = \sqrt{\tilde{\epsilon}_j^2 + \Delta^2}$ are the quasiparticle energies, and $\tilde{\epsilon}_j = \epsilon - \lambda$ are the single-particle energies with respect to the chemical potential λ and Δ is the BCS gap. As usual we have defined $a_{\bar{j}} \equiv a_{j\bar{m}} = (-1)^{j-m} a_{j,-m}$.

For superfluid systems the addition and removal RPA phonons cannot be treated separately. The dispersion relation, that relates the strength of the interaction with the energy roots of the RPA, becomes a two-by-two determinant. From the RPA equations

$$\Gamma_n^\dagger = \sum_j \Big(X_n(j) [\alpha_j^\dagger \alpha_j^\dagger]_{00} + Y_n(j) [\alpha_j \alpha_j]_{00} \Big)\,, \quad (14)$$

$$[H, \Gamma_n^\dagger] = \omega_n \Gamma_n^\dagger, \quad (15)$$

we can obtain the following factors:

$$x = \sum_{j_1 \le j_2} |M(j_1 j_2)|^2 \left[\frac{U_{j_1}^2 U_{j_2}^2}{E_{j_1}+E_{j_2}-\omega_n} + \frac{V_{j_1}^2 V_{j_2}^2}{E_{j_1}+E_{j_2}+\omega_n} \right], \quad (16)$$

$$y = \sum_{j_1 \le j_2} |M(j_1 j_2)|^2 \left[\frac{V_{j_1}^2 V_{j_2}^2}{E_{j_1}+E_{j_2}-\omega_n} + \frac{U_{j_1}^2 U_{j_2}^2}{E_{j_1}+E_{j_2}+\omega_n} \right], \quad (17)$$

$$z = \sum_{j_1 \le j_2} |M(j_1 j_2)|^2 (U_{j_1} V_{j_2} U_{j_2} V_{j_1}) \times \left[\frac{1}{E_{j_1}+E_{j_2}-\omega_n} + \frac{1}{E_{j_1}+E_{j_2}+\omega_n} \right], \quad (18)$$

and the dispersion relation is in this case

$$\begin{vmatrix} (1-4\pi G x)\,, & 4\pi G z \\ 4\pi G z\,, & (1-4\pi G y) \end{vmatrix} = 0. \quad (19)$$

It can be shown that $\omega = 0$ is the solution of this equation and correspond to the Goldstone boson corresponding to the breaking of the number of particle symmetry. Once we have obtained the energies ω_n of the different RPA roots, we can write the components of the RPA phonon in the form

$$X_n(j,j) = \frac{4\pi G M(j,j)}{E_j + E_j - \omega_n} \left(U_j^2 + V_j^2 \frac{4\pi G z}{(1-4\pi G y)} \right) \Lambda_n\,,$$

$$Y_n(j,j) = \frac{4\pi G M(j,j)}{E_j + E_j + \omega_n} \left(U_j^2 \frac{4\pi G z}{(1-4\pi G y)} + V_j^2 \right) \Lambda_n\,, \quad (20)$$

where Λ_n is determined by normalizing the phonon corresponding to the n-th root of the RPA. The normalization condition reads

$$\sum_j [X_n^2(j) - Y_n^2(j)] = 1. \qquad (21)$$

Finally, we can obtain for each state n the pairing strength parameter $\beta_{\rm P}$ with the following formulae:

$$\beta_{\rm P}(2p) = \sum_j \sqrt{2j+1}\langle n|[a_j^\dagger a_j^\dagger]_{00}|0\rangle = \sum_j \sqrt{2j+1}[U_j^2 X_n(j) + V_j^2 Y_n(j)],$$

$$\beta_{\rm P}(2h) = \sum_j \sqrt{2j+1}\langle n|[a_j a_j]_{00}|0\rangle = \sum_j \sqrt{2j+1}[V_j^2 X_n(j) + U_j^2 Y_n(j)]. \qquad (22)$$

From the two equations above one recovers the four contribution to formula (8) by putting $U = 0$ and $V = 1$ when j is below the Fermi level and by putting $U = 1$ and $V = 0$ when j is above. The predictions of the pairing strength distribution for the superfluid system ^{116}Sn are shown in the two panels of fig. 2. For the calculation we have used the single-particle levels from ref. [11]. These last ones have been proved to give good results in BCS calculations using a pairing strength $G = g/A$, where $g \simeq 20\,\text{MeV}$. We assume that the rest of the levels have occupation probability 1 (0) if they are far below (above) the Fermi surface. The change of the single-particle energies around the Fermi surface has been done, in both cases, taking care of keeping the energy centroids of the exchanged levels in the same position. The figure clearly shows the occurence of high-lying strength which can be associated with the giant pairing vibration. Note that, with respect to the case of ^{208}Pb, there is a minor fragmentation of the strength both in the low-lying and in the high-lying energy region.

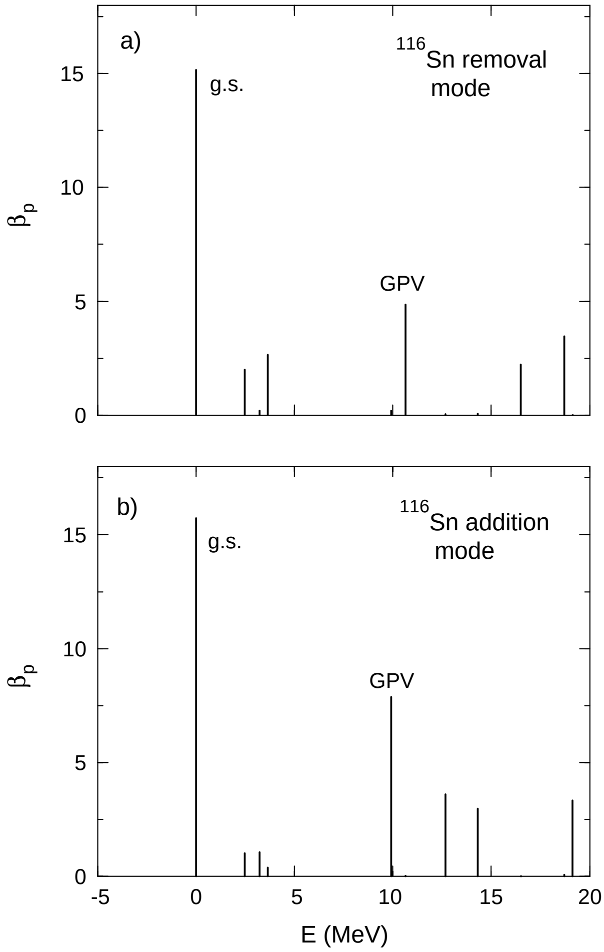

Fig. 2. Pairing response for the removal (a) and addition (b) mode in ^{116}Sn. The ground-state transition and the candidate for GPV are evidenced.

3 Macroscopic form factors for two-particle transfer reactions

The description of the reaction mechanism associated with the transfer of a pair of particles in heavy-ion reactions has always been a rather complex issue. In the limit in which the field responsible for the transfer process is the one-body field generated by one of the partners of the reactions, at least for simple configurations the leading order process is the successive transfer of single particles. In this framework the collective features induced by the pairing interaction arise from the coherence of different paths in the intermediate ($A+1$, $A-1$) channel due to the correlation present in the final ($A+2$) and ($A-2$) states. The actual implementation of such a scheme may turn out not to be a simple task, due to the large number of active intermediate states, and the use of a simpler approach may be desirable. This is offered, for example, by the "macroscopic model" for two-particle transfer reactions, that parallels the formalism used to describe the inelastic excitation of collective surface modes. In that case, as an alternative to the (more correct) microscopic description based on a superposition of particle-hole excitations, one has traditionally resorted to collective form factors of the form [12]

$$F_\lambda(r) = \beta_\lambda R \frac{\mathrm{d}U}{\mathrm{d}r}, \qquad (23)$$

in terms of the radial variation of the ion-ion optical potential U induced by the surface vibrations, with the strength parameter β_λ obtained from the strength of the $B(E\lambda)$ transition. In the case of the pair transfer, the corresponding vibration is the fluctuation of the Fermi surface with respect to the change in the number A of particles, and the corresponding form factor $F_{\rm P}$ is assumed to have the parallel form [5]

$$F_{\rm P}(r) = \beta_{\rm P} \frac{\mathrm{d}U}{\mathrm{d}A}, \qquad (24)$$

in terms of the "pairing deformation" parameter $\beta_{\rm P}$ associated with that particular transition, defined in the previous section. The assumption of simple scaling law between nuclear radius R and mass number A allows to rewrite the two-particle transfer form factor into an expression which is formally equivalent to the one for inelastic excitation, namely

$$F_{\mathrm{P}}(r) = \frac{\beta_{\mathrm{P}}}{3A} R \frac{\mathrm{d}U}{\mathrm{d}r}. \qquad (25)$$

This formalism has been successfully applied to quite a number of two-particle transfer reactions [13,14]. As in the case of inelastic excitations, macroscopic collective form-factors may in some cases only give a rough estimate to the data, requiring more elaborate microscopic descriptions. Nonetheless, the use of simple macroscopic form factors is of unquestionable usefulness in making predictions, in particular in cases, as the one we are discussing, where experimental data are not yet available and estimates are needed in order to plan future experiments.

4 Applications: estimates of two-neutron transfer cross-sections

In order to evidence the possible role of unstable beams in the study of high-lying pairing states, we compare in this section two-particle transfer reactions induced either by a traditionally available beam (*e.g.*, the (^{14}C, ^{12}C)) or by a more exotic beam (*e.g.*, the reaction (^{6}He, ^{4}He)). As a target, we have considered the two cases of ^{208}Pb and ^{116}Sn, as representative cases of normal and superfluid systems in the pairing channels. In both cases, we have considered the full pairing $L = 0$ response, *e.g.*, all transitions to 0^+ states in ^{210}Pb and ^{118}Sn, as described in sect. 2. The Q-values corresponding to the transitions to the ground states and to the GPV states are displayed in table 1. For each considered state the two-particle transfer cross-section has been calculated on the basis of the DWBA (using the code Ptolemy [15]) employing the macroscopic form factor described above, with a strength parameter as resulting from the RPA calculation. For the ion-ion optical potential, the standard parameterization of Akyuz-Winther [16] has been used for the real part, with an imaginary part with the same geometry and half its strength. In all cases, the bombarding energy has been chosen in order to correspond, in the center-of-mass frame, to about 50% over the Coulomb barrier. The angle-integrated $L = 0$ excitation function is shown in fig. 3b as a function of the excitation energy E_x for the ^{208}Pb(^{14}C,^{12}C)^{210}Pb reaction at $E_{\mathrm{cm}} = 95$ MeV. For a more realistic display of the results, the contribution of each discrete RPA state is distributed over a Lorentzian with $\Gamma = kE_x^2$, with k adjusted to yield a width of 4 MeV for the giant pairing vibration. As the figure shows, the large (negative) Q-value associated with the region of the GPV (see table 1) completely damps its contribution, and the excitation function is completely dominated by the transition to the ground state and the other low-lying states. The situation is very different for the ^{208}Pb(^{6}He,^{4}He)^{210}Pb reaction at $E_{\mathrm{cm}} = 41$ MeV, whose excitation function is shown in fig. 3a. In this case the weak-binding nature of ^{6}He projectile leads to a mismatched (positive) Q-value for the

Table 1. Q-values for ground-state and GPV transitions. The target (column) and projectile (row) are specified.

	^{14}C → ^{12}C	^{6}He → ^{4}He
^{116}Sn → ^{118}Sn$_{\mathrm{g.s.}}$	3.15 MeV	15.298 MeV
^{208}Pb → ^{210}Pb$_{\mathrm{g.s.}}$	−4 MeV	8.148 MeV
^{116}Sn → ^{118}Sn$_{\mathrm{GPV}}$	−6.746 MeV	5.402 MeV
^{208}Pb → ^{210}Pb$_{\mathrm{GPV}}$	−15.81 MeV	−3.662 MeV

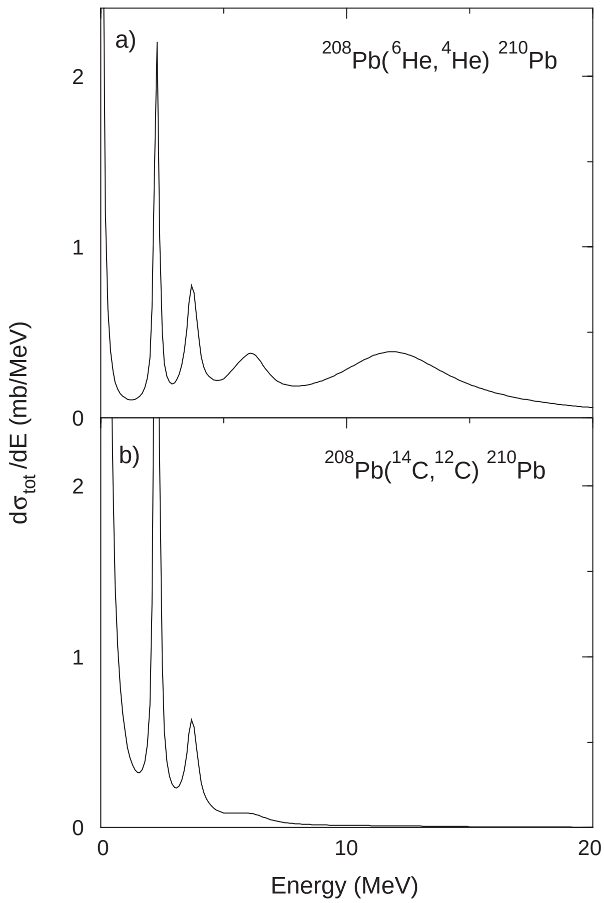

Fig. 3. Differential cross-sections as a function of the excitation energy for the two reactions: a) ^{208}Pb(^{6}He,^{4}He)^{210}Pb, and b) ^{208}Pb(^{14}C,^{12}C)^{210}Pb. See text for details.

ground-state transition ($Q_{\mathrm{g.s.}} = 8.148$ MeV), favouring the transfer process to the high-lying part of the pairing response. In this case the figure shows that, in spite of a smaller pairing matrix element, the transition to the GPV is of the same order of magnitude of the ground-state transfer (1.8 mb for g.s. and 3.1 mb for the GPV). Note that a total cross-section to the GPV region of the order of some millibarn should be accessible with the new large-scale particle-gamma detection systems.

A similar behaviour is obtained in the case of a tin target. In fig. 4 the corresponding excitation functions for the ^{116}Sn(^{14}C,^{12}C)^{118}Sn reaction (at $E_{\mathrm{cm}} = 69$ MeV) and the ^{116}Sn(^{6}He,^{4}He)^{118}Sn reaction (at $E_{\mathrm{cm}} = 40$ MeV) are compared. Now the transition to the GPV dominates over the ground-state transition when using an He beam

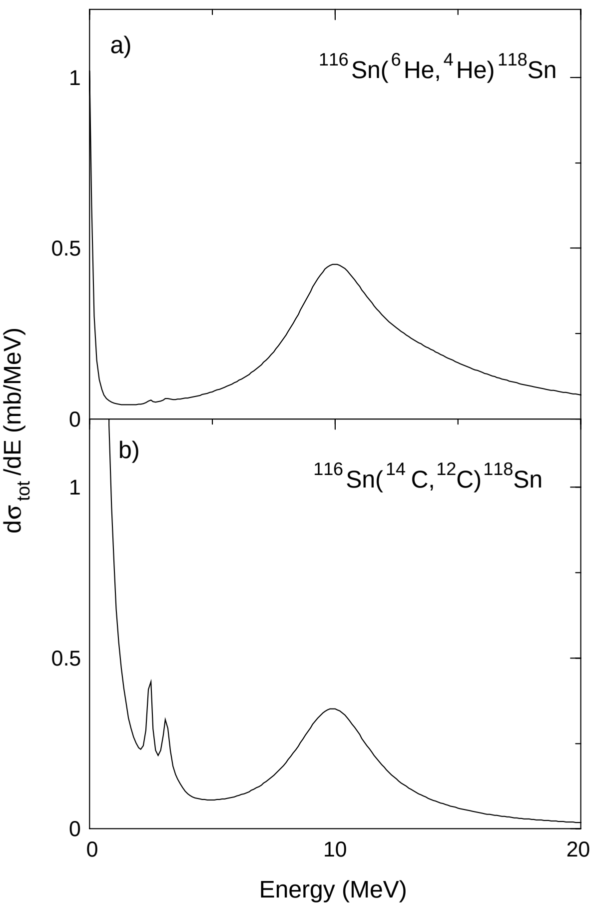

Fig. 4. Differential cross-sections as a function of the excitation energy for the two reactions: a) ^{116}Sn(^{6}He,^{4}He)^{118}Sn, and b) ^{116}Sn(^{14}C,^{12}C)^{118}Sn. The comparison between the GPV and the ground state clearly shows the different strength. Notice the different vertical scale with respect to fig. 3.

(0.4 mb for g.s. and 2.4 mb for the GPV). From a comparison with the RPA strength distributions of figs. 1 and 2 one can see that the giant pairing vibrations is definitely favoured by the use of an ^{6}He beam instead of the more conventional ^{14}C one, because the transition to the ground state is hindered, while the GPV is enhanced (or not changed) because of the effect of the Q-value.

5 Conclusions

The role of radioactive-ion beams for studying different features of the pairing degree of freedom via two-particle transfer reactions is underlined. A ^{6}He beam may allow an experimental study of high-lying collective pairing states, that have been theoretically predicted, but never seen in measured spectra, because of previously unfavourable matching conditions. The modification in the reaction Q-value, when passing from ^{14}C to ^{6}He, that is a direct consequence of the weak-binding nature of the latter neutron-rich nucleus, is the reason of the enhancement of the transition to the giant pairing vibration with respect to the ground state.

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